

Q- The vibrations of a string of length 60 cm fixed at both ends are represented by the equation

$$y = 4 \sin (\pi x/15) \cos (96 \pi t)$$

(here x and y are in cm and t in seconds)

- What is the maximum displacement of a point at x = 5 cm?
- Where are the nodes located along the string?
- What is the velocity of the particle at x = 7.5 cm and at t = 0.25 s?
- Write down the equations of the component waves whose superposition's give the above wave.

y = 4 sin (πx/15) cos (96 πt) can be broken up into

$$y = 2 \left[\sin \left(\frac{\pi x}{15} + 96 \pi t \right) + \sin \left(\frac{\pi x}{15} - 96 \pi t \right) \right]$$

Thus, the waves are of the same amplitude and frequency but travelling in opposite directions which thus, superimpose to give a standing wave.

- (a) At x = 5cm the standing wave equation gives

$$y = 4 \sin (5\pi/15) \cos (96\pi t)$$

$$= 4 \sin (\pi/3) \cos (96 \pi t)$$

$$= 4 \times (1/2) \cos (96 \pi t)$$

∴ Maximum displacement = 2 cm.

- (b) The nodes are the points permanently at rest. Thus, they are those points for which $\sin (\pi t/15) = 0$

Or $(\pi x/15) = n\pi$, where n = 0, 1, 2, 3, 4.....

Or x = 15 n i.e. at x = 0, 15, 30, 45 and 60 cm.

- (c) The particle velocity is given by differentiating equation of the wave partially with respect to time as

$$v = \left(\frac{\partial y}{\partial t} \right) = 4 \sin \left(\frac{\pi x}{15} \right) (96\pi) (-\sin 96\pi t)$$

$$= -384 \pi \sin \left(\frac{\pi x}{15} \right) \sin(96\pi t)$$

at x = 7.5 and t = 0.25 we get

$$v = \left(\frac{\partial y}{\partial t} \right) = -384 \pi \sin \left(\frac{\pi * 7.5}{15} \right) \sin(96\pi * 0.25)$$

$$\text{Or } v = \left(\frac{\partial y}{\partial t} \right) = -384 \pi \sin \left(\frac{\pi}{2} \right) \sin(24\pi) = 0$$

- (d) The equations of the component waves are

(e)

$$y_1 = 2 \sin \left(\frac{\pi x}{15} + 96\pi t \right) \quad \text{and} \quad y_2 = 2 \sin \left(\frac{\pi x}{15} - 96\pi t \right)$$