Q- A car of mass m starts moving such that its velocity varies with distance travelled x as $\boldsymbol{v = k\sqrt{x}}$ where k is a constant. Find the total work performed by all forces acting on the car in time T.

The velocity as a function of x is given by

$$v = k\sqrt{x}$$

Or $$\frac{dx}{dt} = k\sqrt{x}$$

Or $$\frac{dx}{\sqrt{x}} = k\,dt$$

Integrating above equation with proper limits the distance travelled in first t second is given by

$$\int_0^x \frac{dx}{\sqrt{x}} = k\int_0^t dt$$

Or $$2\sqrt{x} = k\,t \qquad \text{--------- (1)}$$

Now the acceleration in terms pf velocity and displacement is given as

$$a = v\,\frac{dv}{dx}$$

Substituting value of v we get

$$a = k\sqrt{x}\,\frac{d}{dx}(k\sqrt{x})$$

Or $$a = k^2\sqrt{x} * \frac{1}{2\sqrt{x}} = \frac{k^2}{2}$$

And hence the net force action on the car will be

$$F = ma = m\,\frac{k^2}{2}$$

Thus the work done on the car in very small displacement in the direction of force will be

$$dW = F.dx = F.v\,dt$$

Substituting value of F and v we get

$$dW = m\frac{k^2}{2}.k\sqrt{x}\,dt = m\frac{k^2}{2}.k\frac{kt}{2}dt \qquad \text{--- using equation (1)}$$

Or $$dW = m\,\frac{k^4}{4}.t\,dt$$

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Integrating above equation we get

$$\int_0^W dW = m\,\frac{k^4}{4}.\int_0^t t\,dt$$

Or $$W = m\,\frac{k^4}{4}.\frac{t^2}{2} = \frac{1}{8}mk^4t^2$$

Hence the work done on the car in first t second is

$$W = \frac{1}{8}mk^4t^2$$

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