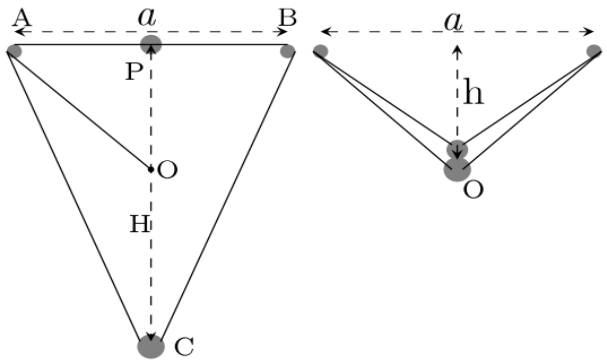
Q An endless light string of length $2l$ is passing over two smooth pegs A and B at a distance a on same horizontal line. Beads of mass m and M are threaded into it and one mass in on each parts between the pegs. The lighter bead of mass m is held at midpoint of AB and released. Find ratio m/M for which the two beads just meets.

When we held the bead of mass m the other is hanging bellow and the string between the pegs through bead of mass m is straight and horizontal its length is a as in figure. The length of the string through M will be $2l$ - a. The depth of M from the pegs can be calculated using triangle APC as



$$H = \sqrt{\left(\frac{2l-a}{2}\right)^2 - \left(\frac{a}{2}\right)^2}$$

The parts of the string are equal in length and thus the depth of O is given by solving triangle APO as

$$h = \sqrt{\left(\frac{l}{2}\right)^2 - \left(\frac{a}{2}\right)^2}$$

If they just meet means their velocities becomes zero at O and posses no kinetic energy. Applying law of conservation of energy we get

Loss in potential energy of m = gain in PE of M

Or $mgh = Mg(H-h)$

Or $(m+M)h = MH$

Substituting values of h and H we get

$$(m+M)\sqrt{\left(\frac{l}{2}\right)^2 - \left(\frac{a}{2}\right)^2} = M\sqrt{\left(\frac{2l-a}{2}\right)^2 - \left(\frac{a}{2}\right)^2}$$

Or $\quad (m+M) = M\sqrt{\dfrac{(2l-a)^2-a^2}{l^2-a^2}}$

Or $\quad \dfrac{m}{M}+1 = \sqrt{\dfrac{4l^2-4la}{l^2-a^2}}$

Or $\quad \dfrac{m}{M} = 2\sqrt{\dfrac{l(l-a)}{l^2-a^2}} - 1$

Or $\quad \dfrac{m}{M} = 2\sqrt{\dfrac{l}{l+a}} - 1$