Q- Two blocks of mass m and M are attached to the two ends of a light spring of constant k. The system is held by m such that M is suspended through the spring in air. Now the system is lowered slowly such that M just touches the tabletop and m is released. Find the maximum displacement of m before it comes to instantaneous rest.

When the block M just touches the table top its weight is balanced by the tension in the spring and thus elongation Δl_1 in the spring is given by

$$\Delta l_1 = \frac{Mg}{k}$$

As we release the block of mass m, due to gravity and spring force it comes down and its velocity increases till the spring gets its natural length. It still comes down due to its weight till it comes to rest. Let the compression in the spring in this situation is Δl_2 which is given by law of conservation of energy as

Gain in elastic potential energy = loss in gravitational potential energy

Or $$\frac{1}{2}\, k\, (\Delta l_2^2 - \Delta l_1^2) = mg(\Delta l_2 + \Delta l_1)$$

Or $$\Delta l_2^2 - \left(\frac{Mg}{k}\right)^2 = \frac{2mg}{k}\left(\Delta l_2 + \frac{Mg}{k}\right)$$ (substituting for Δl_1)

Or $$\Delta l_2^2 - \frac{2mg}{k}\Delta l_2 - \left(\frac{Mg}{k}\right)^2 - \frac{2mMg^2}{k^2} = 0$$

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Or $$\Delta l_2 = \frac{mg}{k} \pm \sqrt{\frac{m^2g^2}{k^2} + \left(\frac{Mg}{k}\right)^2 + \frac{2mMg^2}{k^2}}$$

Or $$\Delta l_2 = \frac{mg}{k} \pm (M+m)\frac{g}{k}$$

Hence the total displacement of the block of mass m will be

$$\Delta L = \Delta l_1 + \Delta l_2 = \frac{Mg}{k} + \frac{mg}{k} \pm (M+m)\frac{g}{k}$$

Or $$\Delta L = 2(M+m)\frac{g}{k}$$ downwards (negative sign is inappropreate)