Q- A thin ring of mass m and radius R hangs freely about a point on its periphery. It oscillates by a small angle about point of suspension.
(a) Along its plane
(b) Perpendicular to its plane. Find the time period in both cases.

If a particle of mass m is moving to and fro on a straight line and the restoring force acting on it is given by F = - kx, (k is a constant known as force constant) the motion is called simple harmonic and its time period of such oscillations is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Similarly, if a body oscillates about an axis and the restoring torque is given by τ = - k θ, where k is the torque constant and θ is angular displacement, its motion is called rotational simple harmonic motion and the time period is given by

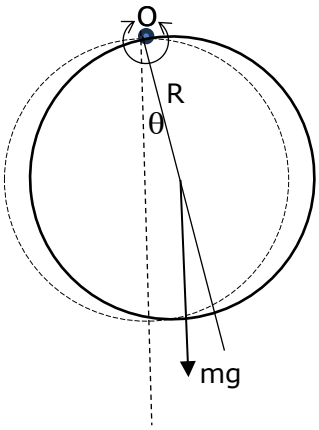
$$T = 2\pi\sqrt{\frac{I}{k}}$$

Here I is the moment of inertia of the bout the axis of rotation.

(a) When the ring is making an angle θ from its the torque on it is due to its weight and is given by

$$\tau = -mg\,R\sin\theta$$

(here negative sign shows that it is restoring torque tries to bring the ring back to its equilibrium position)



If θ is very small and measured in radians $\sin\theta \cong \theta$ and we can write

$$\tau = -mg\,R\,\theta$$

Or $$\tau = -k\,\theta$$

Here the constant k = mgR and the motion of the rotational simple harmonic.

Now for the moment of inertia of the ring about the axis passing through point of suspension O will be given by using parallel axis theorem as

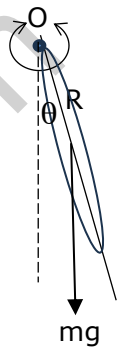
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$$I = I_c + md^2 = mR^2 + mR^2 = 2mR^2$$

And thus the time period for the small oscillation will be

$$T = 2\pi\sqrt{\frac{I}{k}} = 2\pi\sqrt{\frac{2mR^2}{mgR}} = 2\pi\sqrt{\frac{2R}{g}}$$

(b) In this case the ring rotates about its tangent passing throw O. The restoring torque and the constant k are same as in previous case but the moment of inertia will be different. Here we use perpendicular axis theorem first and then the parallel axis theorem.



The moment of inertia of the ring about its diameter is given by using perpendicular axis theorem as

$$I = I_d + I_d$$

Or $$mR^2 = 2I_d$$

Gives $$I_d = \frac{1}{2}mR^2$$

Now using parallel axis theorem the moment of inertia about tangent will be

$$I_T = I_d + mR^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$$

And thus the time period for small oscillation about tangent of the ring will be

$$T = 2\pi\sqrt{\frac{I}{k}} = 2\pi\sqrt{\frac{1.5\,mR^2}{mgR}} = 2\pi\sqrt{\frac{3R}{2g}}$$

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