

Q- A solid right circular cylinder of mass 5kg and diameter 12cm is suspended by a spring of constant $K = 2 \text{ N/cm}^{-1}$ and hangs partially submerged in a large water body with its axis vertical. Find the period of small vertical oscillations. Neglect viscosity and surface tension of water.

The cylinder is in equilibrium means all the forces are balanced.

If from this position we displace cylinder with a small distance Δy vertically downwards then

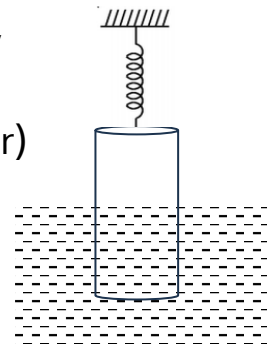
1. The spring gets elongated further by Δy and thus an additional force $K \Delta y$ will act in upward direction.
2. An additional volume of cylinder equal to $A \Delta y$ will submerge in the water due to which an additional upthrust equal to the weight of displaced water will act on the in upward direction.

Hence the net restoring force on the cylinder will be given by

$$F = -(K \Delta y + A \Delta y \rho g) \text{ (Here } \rho \text{ is the density of water)}$$

$$\text{Or } F = -(K + A \rho g) \Delta y$$

$$\text{Or } F \propto -\Delta y$$



The constant of proportionality is $K + A \rho g$ and hence the Period of small vertical oscillation will be given by

$$T = 2\pi \sqrt{\frac{m}{K + A \rho g}} = 2\pi \sqrt{\frac{m}{K + \pi r^2 \rho g}}$$

$$\text{Or } T = 2 * 3.14 \sqrt{\frac{5}{200 + 3.14 * (0.06)^2 * 1000 * 9.8}} = 0.8 \text{ s}$$