Q- A light platform is attached to the top end of a vertical spring of force constant K. The lower end of the spring is fixed to the ground. A small ball of mass *m* is dropped from height *h* above the platform and strikes it elastically. Find the maximum velocity and maximum acceleration of the ball after it strike the platform.

Solution:

As the net force on the mass after contact with spring is directly proportional to x, and in the direction opposite to the compression of the spring, the motion of the mass is simple harmonic.

In SHM the velocity of the particle is maximum at equilibrium position which is given by

$$Kx_0 - mg = 0$$

Here x_0 is the compression in the spring, thus

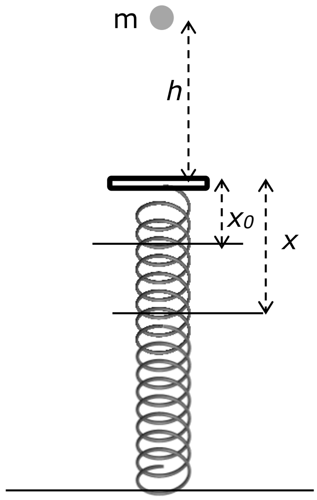
$$x_0 = \frac{mg}{K} \qquad \text{-----------} (1)$$

Mass of the platform is negligible and thus the mass will continue with the same velocity after strike with it and thus according to the velocity at the equilibrium position which is maximum is given by law of conservation of energy as

Loss in gravitational PE = gain in elastic PE + gain in KE

Or $$mg\,(h + x_0) = \frac{1}{2}\,Kx_0^2 + \frac{1}{2}m\,v_{max}^2$$

Or $$v_{max}^2 = 2g\,(h + x_0) - \frac{K}{m}x_0^2$$



Substituting value of x_0 we get

$$v_{max}^2 = 2g\left(h + \frac{mg}{K}\right) - \frac{K}{m}\left(\frac{mg}{K}\right)^2$$

Or $$v_{max}^2 = 2g\,h + 2\frac{mg}{K} - \frac{mg}{K}$$

Or $$v_{max}^2 = g\left(2h + \frac{mg}{K}\right)$$

Or $$v_{max} = \sqrt{g\left(2h + \frac{mg}{K}\right)}$$

In SHM the Acceleration is maximum at the extreme positions where the displacement from the equilibrium position is maximum, called amplitude and the velocity is zero. If the total compression in the spring is x than using the conversion of energy we can write

Loss in gravitational potential energy = gain in elastic potential energy

Or $mg(h+x) = \frac{1}{2} K\,(x)^2$

Or $K\,(x)^2 - 2mgx - 2mgh = 0$

Gives $x = \frac{2mg \pm \sqrt{4m^2g^2 + 8K\,mgh}}{2K}$

Or $x = \frac{mg}{K} + \frac{\sqrt{m^2g^2 + 2K\,mgh}}{K}$ (x can't be negative hence + only)

Now the force on m at this position is F = Kx − mg in upward direction hence maximum acceleration is given by

$$a_{max} = \frac{F}{m} = \frac{Kx}{m} - g$$

Substituting for x we get

$$a_{max} = \frac{K}{m}\left(\frac{mg}{K} + \frac{\sqrt{m^2g^2 + 2K\,mgh}}{K}\right) - g$$

Or $a_{max} = g + \left(\frac{K}{m}\frac{\sqrt{m^2g^2 + 2K\,mgh}}{K}\right) - g$

Or $a_{max} = \left(\frac{\sqrt{m^2g^2 + 2K\,mgh}}{m}\right)$

Or $a_{max} = g\left(\sqrt{1 + \frac{2Kh}{mg}}\right)$