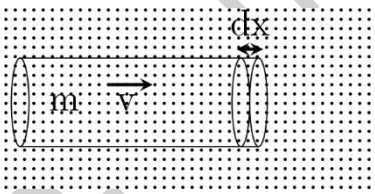
Q- A solid cylinder, of radius c acted upon by no forces, moves parallel to its axis through a uniform cloud of fine dust, of volume density ρ which is at rest. If the particles of dust which meet the cylinder adhere to it, and if m_o and u be the mass and velocity at the beginning of the motion, find the distance x traversed in time t by the cylinder.

Mass of the system moving at any time t is the mass of the cylinder and the mass of the dust deposited on the front plane surface of the cylinder therefore, this is the case of variable mass system.

If the distance moved by the cylinder in time t be x than volume of the cloud swept by the cylinder will be face area of the cylinder A *x and the mass added will be Axρ. Thus, the mass of the system as a function of time is given by



$$m = m_0 + Ax\rho = m_0 + \pi c^2 x\rho$$

If at time t, in an infinitesimal time dt it covers a distance dx (velocity dx/dt), according to law of conservation of linear momentum

$$m_o u = m\,\frac{dx}{dt}$$

Or $$m_o u = (m_0 + \pi c^2 x\rho)\,\frac{dx}{dt}$$

Or $$(m_0 + \pi c^2 x\rho)dx = m_o u\,dt$$

Integrating this

$$\int_0^x (m_0 + \pi c^2 x\rho)dx = \int_0^t m_o u\,dt$$

Or $$m_0 x + \frac{1}{2}\pi c^2 x^2 \rho = m_o u\,t$$

Or $$\pi c^2 x^2 \rho + 2m_0 x - 2m_o u\,t = 0$$

Or $$x = \frac{-2m_0 \pm \sqrt{4m_0^2 - 8\pi c^2 \rho(-2m_0 ut)}}{2\,\pi c^2 \rho}$$

Or $$x = \frac{-m_0 \pm \sqrt{m_0^2 + 2m_0 u\pi c^2 \rho\,t}}{\pi c^2 \rho}$$

Or $$x = \frac{\sqrt{m_0^2 + 2m_0 u\pi c^2 \rho\,t} - m_0}{\pi c^2 \rho}$$

(As x cannot be negative.)