Q-A particle is projected in a vertical plane from a point 0 with velocity u at angle α to the horizontal. Find the time at which the particle is moving at angle θ to the horizontal.

The horizontal component of velocity 'u cos α' remain constant in a projectile motion as the only force considered is the gravity. This means the direction of motion of the particle is changing only to change in its vertical component of its velocity.

Initially the vertical component of velocity of particle is u sin α hence after time t its vertical component is given by using first equation of motion [v = u + at] as

$v' = u \sin \alpha + (-g)\, t$ (here t is acceleration due to gravity, downwards)

Now if the direction of velocity of the projectile is making angle θ with the horizontal after time t we get

$$\tan\theta = \frac{v'}{u\cos\alpha}$$

Or $\tan\theta = \frac{u\sin\alpha - gt}{u\cos\alpha}$

Or $\tan\theta\, u\cos\alpha = u\sin\alpha - gt$

Or $gt = u\sin\alpha - \tan\theta\, u\cos\alpha$

Or $t = \frac{u\cos\alpha(\tan\alpha - \tan\theta)}{g}$

This is the time after projection the velocity of particle makes angle θ to the horizontal.