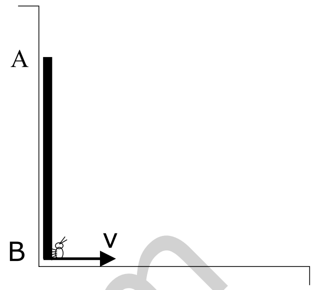
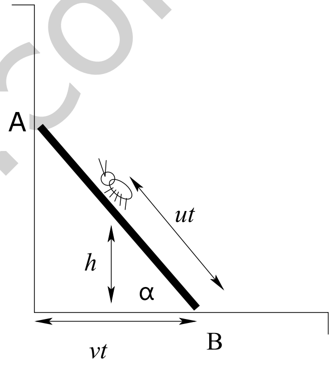
The bug climber

Q- A stick of length L is placed vertically by the wall. At its lower end sits a bug. The end B of the stick starts moving to the right with constant speed v, and at the same moment the bug starts crawling along the stick with speed u relative to the stick. What is the maximam height above the floor that the bug reaches while it crawls along the stick? End A of the stick does not lose contact with the wall.



At time t, lower end of the stick will be at a distance $v*t$ from the wall, making angle α such that cos α = $v*t/L$ with the floor, and the bug progressed to a point $u*t$ from this end (along the stick).



The height of the bug above the floor will be

$$y = ut\sin\alpha = ut\sqrt{1-\cos^2\alpha}$$

Or $$y = ut\sqrt{1-\left(\frac{vt}{L}\right)^2} = \frac{ut}{L}\sqrt{L^2-v^2t^2} \quad \text{--------- (1)}$$

Form maximum height dy/dy should be equal to zero, thus differentiating above equation we get

$$\frac{dy}{dt} = \frac{u}{L}\left[t\,\frac{-v^2 2t}{2\sqrt{L^2-v^2t^2}} + 1.\sqrt{L^2-v^2t^2}\right] = 0$$

Or $$\frac{u}{L}\left[\frac{-v^2t^2 + L^2 - v^2t^2}{\sqrt{L^2-v^2t^2}}\right] = 0$$

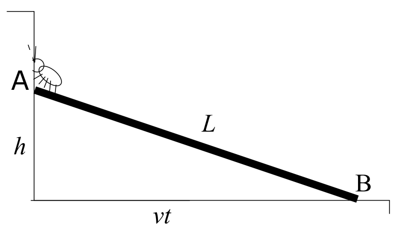
Or $$L^2 - 2v^2t^2 = 0$$

Or $$t = \frac{L}{v\sqrt{2}}$$

Substituting this value of t in equation (1)

$$y_{max} = \frac{u}{L} * \frac{L}{v\sqrt{2}}\sqrt{L^2-\frac{L^2}{2}} = \frac{uL}{2v}$$

If u > 2v, y_{max} > L which is not possible because the bug will reach A before the stick slip to ground and in this case maximum height is the height of A when the bug reaches A or t = L/u



Then $$y_{max} = \sqrt{L^2-v^2t^2} = \sqrt{L^2-v^2\left(\frac{L}{u}\right)^2}$$

Or $$y_{max} = L\sqrt{1-\frac{v^2}{u^2}}$$

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