

Q- Three particles, located initially on the vertices of an equilateral triangle of side L, start moving with a constant tangential acceleration 'a' towards each other in a cyclic manner, forming spiral loci that converge at the centroid of the triangle. Find the length of one such spiral.

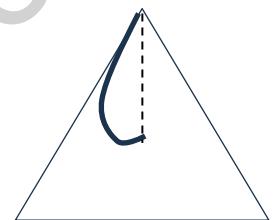
The particles are directed toward each other with the same velocity symmetrically so forming equilateral triangle all the time. The side of the triangle reduces and the orientation changes continuously till they converge at the centroid.

The component of their acceleration $a \cos 30^\circ = a\sqrt{3}/2$ (towards the centroid will bring them closure while the other component will rotate them on the instantaneous circular path. As the distance between the vertex and centroid is $(L/\sqrt{3})$ the time t in which they will converge will be given by

$$[s = ut + \frac{1}{2} a t^2]$$

$$\frac{L}{\sqrt{3}} = 0 + \frac{1}{2} \frac{a\sqrt{3}}{2} t^2$$

$$\text{Or } t = \sqrt{\frac{4L}{3a}}$$



The distance covered by the particle i.e. length of the spiral is due to tangential acceleration and thus given by

$$s = ut + \frac{1}{2} a t^2$$

$$l = 0 + \frac{1}{2} a t^2 = \frac{1}{2} a \frac{4L}{3a} = \frac{2}{3} L$$