Q- An iPod is left running inside a thin insulating case 2.5 mm thick with thermal conductivity of 0.02 W/K/m The case is black and at 25 degrees Celsius and exchanges energy with it's surroundings, all of which are at 20 degrees Celsius, by radiation alone. What is the temperature inside the case?

Solution:
The radiant power (amount of energy radiated per unit time) from the surface of a black body is given by Stefan's law as

$$\frac{dQ}{dt} = A\sigma(T^4 - T_0^4) \qquad \text{---------- (1)}$$

Where A is the surface area, σ is Stefan constant, T is the absolute temperature of the body and T_0 is the absolute temperature of the surroundings.
(As the surface of the case is black and the emissivity is not given, can be considered as black body).

This energy is coming from the inside of case and is coming out by conduction. The rate of heat conduction is given by

$$\frac{dQ}{dt} = \frac{KA(t_1 - t_2)}{d} \qquad \text{---------- (2)}$$

Where K is the conductivity of the material, A is the area of cross section $t_1 - t_2$ is the temperature difference between the two ends and d is the thickness of the sheet of length of the rod.

Now the heat conducted through the sheet of case is only radiated from its surface hence the rate of heat conducted to the surface will be equal to the heat radiated from its surface to the surrounding.
Thus from equations (1) and (2) we get

$$A\sigma(T^4 - T_0^4) = \frac{KA(t_1 - t_2)}{d}$$

Or $$\sigma(T^4 - T_0^4) = \frac{K(t_1 - t_2)}{d}$$

Gives $$t_1 = t_2 + \frac{\sigma(T^4 - T_0^4)d}{K} \qquad \text{---------- (3)}$$

Now here

	Inside temperature of the case	t_1 =?
	Outside temperature of the case	$t_2 = 25^0$ C
	Absolute temperature of the surface	T_1 = 273 + 25 = 298 K
	Absolute temperature of the surrounding	T_2 = 273 + 20 = 293 K
	Conductivity of the material	K = 0.02 W K^{-1} m^{-1}
	Thickness of the case	d = 2.5 mm = $2.5*10^{-3}$ m
And	Stefan constant	σ = 5.67×10^{-8} W·m^{-2}·K^{-4}

Substituting the values in equation (3) the temperature of the inner surface of the case is given as

$$t_1 = t_2 + \frac{\sigma(T^4 - T_0^4)d}{K}$$

Or $$t_1 = 25 + \frac{5.76*10^{-8}(298^4 - 293^4)*2.5*10^{-3}}{0.02} = 28.66^0 C$$