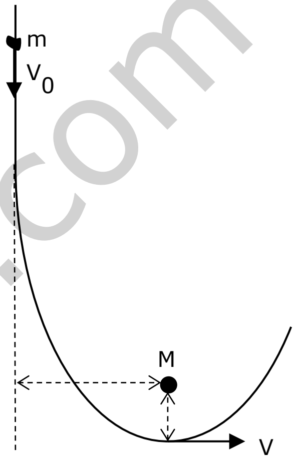
Q- A cosmic body is approaching the earth with velocity v_0 and the perpendicular distance of the centre of the earth from the line of motion of the cosmic body when it is in free space (aiming parameter) is l. Find the minimum distance of the body from the centre of earth if it passes through the earth without impact. (Neglect forces other than earth' s gravity.)

Let the mass of the body is m and that of earth is M. At large distance the body is moving freely with velocity v_0.



As the body approaches near the earth, due to gravitational attraction of the earth its direction changes and at some time itis nearest to the earth. The minimum distance depends on its velocity in free space and the aiming parameter. When it is at minimum distance from the centre of earth, its velocity will be perpendicular to the line joining the centre of the earth and the body.

If we considering earth and body as a system, as there no external force or torque acting on the system angular momentum of the body about the centre of the earth will be

$$\mathrm{L} = \text{momentum} * \text{ arm length}$$

Or $$\mathrm{L} = \mathrm{mv}_0 * l = m\,v * r$$

Or $$\mathrm{v} = \frac{\mathrm{v}_0 l}{r} \quad \text{----------------} \quad (1)$$ (here r is the minimum distance)

As there is no nonconservative force is acting the energy of the system remains conserved thus we get

$$\frac{1}{2}mv_0^2 = \frac{1}{2}\,mv^2 + \left(-\frac{GMm}{r}\right)$$

Or $$v_0^2 = v^2 - \left(\frac{2GM}{r}\right)$$

Substituting v from equation (1) we get

$$v_0^2 = \left(\frac{v_0 l}{r}\right)^2 - \left(\frac{2GM}{r}\right)$$

Or $$r^2 v_0^2 = (v_0 l)^2 - (2GMr)$$

Or $$v_0^2\,r^2 + 2GM\,r - (v_0 l)^2 = 0$$

Or $$r = \frac{-2GM \pm \sqrt{4G^2M^2 + 4v_0^4 l^2}}{2v_0^2}$$

Or $$r = \frac{GM}{v_0^2}\left[-1 \pm \sqrt{1 + \frac{v_0^4 l^2}{G^2M^2}}\right]$$

Or $$r = \frac{GM}{v_0^2}\left[-1 + \sqrt{1 + \frac{v_0^4 l^2}{G^2M^2}}\right]$$ (r to be positive thus positive sign)