Q- A cylindrical tank of cross-sectional area A filled with a liquid to a height H. The tank has a small hole at its base of area a (a < < A). Find the time during which the liquid level falls to a height h = H/n (where n > 1).

Let y be the instantaneous height of liquid in the tank than, the instantaneous velocity of flow through the hole is given by using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

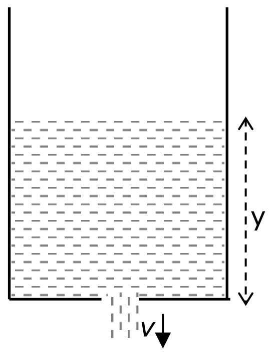
Here P is the pressure, v is velocity, r is the density of fluid and h is its height.

As the area of cross-section at the top is much larger, at the top velocity of liquid is negligible, height is y and pressure is atmospheric pressure. Just outside of the bottom the velocity is say v, height is zero and the pressure is atmospheric pressure thus

$$P_a + \frac{1}{2}\rho * 0 + \rho g y = P_a + \frac{1}{2}\rho v^2 + \rho g * 0$$

gives $v = \sqrt{2gy}$

Now let in a small time dt the level of liquid in the container falls by dy than the velocity of the top level will be – dy/dt and thus using the equation of continuity $(A_1 v_1 = A_2 v_2)$, we have



$$A\left(-\frac{dy}{dt}\right) = a\, v$$

Or $$A\left(-\frac{dy}{dt}\right) = a\sqrt{2gy}$$

Or $$-A\left(\frac{dy}{\sqrt{2gy}}\right) = a\, dt$$

Integrating above we get

$$-A\int_H^h \frac{dy}{\sqrt{2gy}} = a\int_0^t dt$$

Or $$-\frac{A}{\sqrt{2g}}\int_H^h \frac{dy}{\sqrt{y}} = a\, t$$

Or $$-\frac{A}{\sqrt{2g}}\left[2(\sqrt{h} - \sqrt{H})\right] = a\, t$$

Or $$t = \frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{H} - \sqrt{h}\right) = \frac{A}{a}\sqrt{\frac{2H}{g}}\left(1 - \frac{1}{\sqrt{n}}\right)$$