

Q- A cuboidal container with base area A is filled with water up to a height h . Small iron balls are dropped in the container till the upper layer of balls reaches the water surface. If the average density of the content in the container becomes ρ , what is the final height H of water level in terms of densities of water and iron.

Let the density of water be ρ_w and that of iron be ρ_i .

Volume of water in the container will be $V_w = Ah$
 And thus, mass of water in the container will be $m_w = Ah \rho_w$.

Similarly, Volume of iron in the container will be $V_i = A(H-h)$
 And therefore, mass of iron in the container will be $m_i = A(H-h) \rho_i$.

The average density of the material will be

$$\rho = \frac{\text{Total mass}}{\text{Total volume}}$$

$$\text{Or } \rho = \frac{Ah\rho_w + A(H-h)\rho_i}{Ah + A(H-h)}$$

$$\text{Or } \rho = \frac{h\rho_w + (H-h)\rho_i}{h + (H-h)}$$

$$\text{Or } \rho[h + (H - h)] = h\rho_w + (H - h)\rho_i$$

$$\text{Or } \rho H = h\rho_w + H\rho_i - h\rho_i$$

$$\text{Or } H = \left(\frac{\rho_i - \rho_w}{\rho_i - \rho} \right) h$$