

Q- A cuboidal container with base area A is filled with water up to a height h. Small iron balls are dropped in the container till the upper layer of balls reaches the water surface. If the average density of the content in the container becomes ρ , what is the final height H of water level in terms of densities of water and iron.

Let the density of water be ρ_w and that of iron be ρ_i .

Volume of water in the container will be $V_w = Ah$
And thus, mass of water in the container will be $m_w = Ah \rho_w$.

Similarly, Volume of iron in the container will be $V_i = A(H-h)$
And therefore, mass of iron in the container will be $m_i = A(H-h) \rho_i$.

The average density of the material will be

$$\rho = \frac{\text{Total mass}}{\text{Total volume}}$$

$$\text{Or } \rho = \frac{Ah\rho_w + A(H-h)\rho_i}{Ah + A(H-h)}$$

$$\text{Or } \rho = \frac{h\rho_w + (H-h)\rho_i}{h + (H-h)}$$

$$\text{Or } \rho[h + (H - h)] = h\rho_w + (H - h)\rho_i$$

$$\text{Or } \rho H = h\rho_w + H\rho_i - h\rho_i$$

$$\text{Or } H = \left(\frac{\rho_i - \rho_w}{\rho_i - \rho} \right) h$$