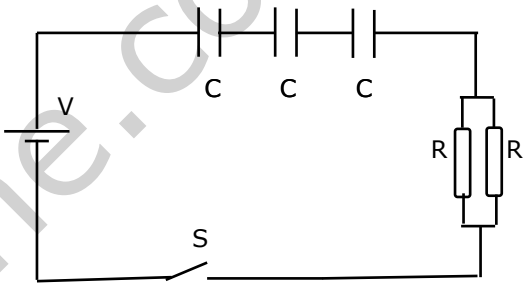
Q- A circuit contains a parallel combination of two resistors of resistance R each and three capacitors with capacitance C each in series with a V volt battery. Initially there is no charge on any capacitor. The switch is closed at t = 0.

(a) What is the charge on the first capacitor plate connected directly to the positive terminal of battery, long time after the switch is closed?

(b) What is the time constant of the circuit?

(c) Find current through the battery as a function of time?



The equivalent capacitance of the three capacitors in series is given by

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$$

Or $C_{eq} = \frac{C}{3}$ ---------- (1)

And the equivalent resistance of the two resistances in parallel is given by

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$$

Or $R_{eq} = \frac{R}{2}$ ---------- (2)

(a) After a long period of time the capacitors will get fully charged and the whole potential is dropped across the capacitor combination and there will be no current in the circuit.

The charge on the capacitor combination is given by

$$Q = C_{eq} * V = \frac{CV}{3}$$

Now as in series combination of the capacitor charge on all capacitors remains same, the charge on the plate nearest to the positive terminal of battery will be

$$Q = +\frac{CV}{3}$$

(b)

The time constant of a R-C circuit is given by

$$\tau = CR$$

Hence the time constant of the given circuit is given by

$$\tau = C_{eq}R_{eq} = \frac{C}{3} * \frac{R}{2} = \frac{CR}{6}$$

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(c)

Let at time t the charge on the capacitors is q and the current in the circuit is I = dq/dt.

The potential drop across the capacitors will be

$$V_C = \frac{q}{C/3} = \frac{3q}{C}$$

And the potential drop across the resistors will be at this time

$$V_R = \frac{IR}{2} = \frac{R}{2}\frac{dq}{dt}$$

Now using the loop rule that the net potential drop throughout the loop is zero, we get

$$V = V_C + V_R$$

Or $V = \frac{3q}{C} + \frac{R}{2}\frac{dq}{dt}$

Or $V - \frac{3q}{C} = \frac{R}{2}\frac{dq}{dt}$

Rearranging the terms we get

$$\frac{dq}{V-\frac{3q}{C}} = \frac{2}{R}\,dt$$

Or $\frac{dq}{VC-3q} = \frac{2}{CR}\,dt$

Integrating the equation within proper limits the charge on the capacitors at time t is given by

$$\int_0^q \frac{dq}{VC-3q} = \frac{2}{CR}\int_0^t dt$$

Or $\left[-\frac{1}{3}\ln(VC-3q)\right]_0^q = \frac{2t}{CR}$

Or $\ln(VC-3q) - \ln VC = -\frac{6t}{CR}$

Or $\ln\left(\frac{VC-3q}{VC}\right) = -\frac{6t}{CR}$

Or $\frac{VC-3q}{VC} = e^{-\frac{6t}{CR}}$

Or $1 - \frac{3q}{VC} = e^{-\frac{6t}{CR}}$

Gives $q = \frac{VC}{3}\left(1 - e^{-\frac{6t}{CR}}\right)$

This is the expression for charge on the capacitors as a function of time t.

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Thus the current in the circuit (charging current) is given by

$$I = \frac{dq}{dt} = \frac{d}{dt}\left[\frac{VC}{3}\left(1 - e^{-\frac{6t}{CR}}\right)\right]$$

Or $$I = \frac{VC}{3}\left[\left(0 - \left(-\frac{6}{CR}\right)e^{-\frac{6t}{CR}}\right)\right]$$

Or $$I = \frac{2\,V}{R}\,e^{-\frac{6t}{CR}}$$

This is the current in the circuit as a function of time.

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