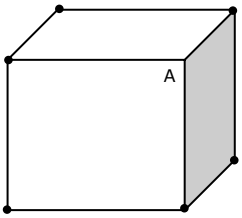
Q- A point charge q is placed at each corner of a cube of edge *l*. Find electrostatic energy of the system of charges.



The total electrostatic energy of a system of charges is given by

$$U = \frac{1}{2}\sum q_i V_i$$

Where q_i is i^{th} charge and V_i is the potential at the point of that charge due to all other charges.

Here we consider point A.

Potential at point A due to a charge on nearest corner (distance '*l*') is given by

$$V_1 = \frac{q}{4\pi\epsilon_0 l} \quad \text{---------- (1)}$$

Potential at A due to a charge on a corner opposite to the face diagonal (distance $\sqrt{2}\,l$) will be given by

$$V_2 = \frac{q}{4\pi\epsilon_0\sqrt{2}\,l} \quad \text{---------- (2)}$$

And potential due to charge on the corner opposite to body diagonal (distance $\sqrt{3}\,l$) will be given by

$$V_3 = \frac{q}{4\pi\epsilon_0\sqrt{3}\,l} \quad \text{---------- (3)}$$

Now the total potential at Point A is due to three charges on nearest corners, three charges on corner opposite to face diagonal and one charge on the corner opposite to the body diagonal of the cube and hence

$$V = 3V_1 + 3V_2 + V_3$$

Or $$V = \frac{3q}{4\pi\epsilon_0 l} + \frac{3q}{4\pi\epsilon_0\sqrt{2}\,l} + \frac{q}{4\pi\epsilon_0\sqrt{3}\,l}$$

Or $$V = \frac{q}{4\pi\epsilon_0 l}\left[3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right]$$

As the system is symmetric, potential at every corner of the cube due to charges on the all other seven corners will be same V and hence the total electrostatic energy of the system is given by

$$U = \frac{1}{2}\sum q_i V_i = \frac{1}{2} * 8 * q * V$$

Or $$U = \frac{1}{2} * 8 * q * \frac{q}{4\pi\epsilon_0 l}\left[3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right]$$

Or $$U = \frac{q^2}{\pi\epsilon_0 l}\left[3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right]$$

This is the required expression.