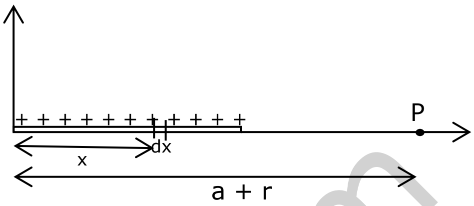
Q- Positive charge Q is distributed uniformly along the x-axis from x = 0 to x = a. A positive point charge q is located at P, on the positive x-axis at x = a + r. (a) Calculate the x-component of the electric field produced by the charge distribution Q at points on the positive x-axis where x > a. (b) Calculate the force that the charge distribution Q exerts on q.

Solution:



(a) Consider a small element of charge dQ on the wire of length dx at distance x from the origin.

Charge Q is distributed uniformly over a length a, hence the charge per unit length will be

$$\lambda = Q/a$$

And the charge on the small element of length dx will be

$$dQ = \lambda * dx = \frac{Q*dx}{a}$$

The electric field strength E at any point P in an electric field is the force per unit test charge hence using Coulomb's law the magnitude of the field strength due to infinitesimal charge element dQ at any point P on x axis, at a distance (a + r) from origin is given by

$$dE = \frac{\lambda * dx}{4\pi\epsilon_0(a+r-x)^2}$$

As the field is along the line joining the charge to the point this field is along + x direction. Hence the field strength due to the whole charge distribution is given by

$$E = \int dE = \int_0^a \frac{\lambda * dx}{4\pi\epsilon_0(a+r-x)^2}$$

Put a + r- x = u
Diff. we get (− dx) = du
x = 0 gives u = a + r
x = a gives u = r

Substituting we get

$$E = -\frac{\lambda}{4\pi\epsilon_0}\int_{a+r}^{r}\frac{du}{(u)^2}$$

Or $$E = -\frac{\lambda}{4\pi\epsilon_0}\left[-\frac{1}{u}\right]_{a+r}^{r}$$

Or $$E = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{a+r}\right]$$

Or $$E = \frac{\lambda a}{4\pi\epsilon_0 r(a+r)}$$

Or $$E = \frac{Q}{4\pi\epsilon_0 r(a+r)}$$ ------------------------- (1)

As the whole charge Q is distributed along x axis the direction of field at P due to each elementary charge is along x axis hence the x component or the resultant field is

$$E_x = \frac{Q}{4\pi\epsilon_0 r(a+r)}$$

(b) As the force on a positive charge due to the field is in the direction of the field, the force on q at P will be along positive x direction.

The magnitude of the force on charge q at point P is given by

$$F = qE = \frac{q*Q}{4\pi\epsilon_0 r(a+r)}$$

Now if r >> a, neglecting a added to r in the expression we get

$$F = \frac{q*Q}{4\pi\epsilon_0 r^2}$$

The force is jest same as due to a point charge Q. it is so because at large distance from the charge distributed over a small line, behaves as a point charge. The flux behaves like they feel transverse repulsion and hence at large distances from any charge distribution they try to become spherically symmetric.