Q- The thickness h of a puddle of water on a waxy surface depends on the density ρ of the liquid, the surface tension γ and acceleration due to gravity g. Find a relationship between the thickness and the other 3 variables.

Let the thickness of the puddle h depends on the quantities as given by the relations as

$$h \propto \rho^a \gamma^b g^c$$

Where a b and c are positive numbers

Or $h = K\, \rho^a \gamma^b g^c$ --------------------------- (1)

Here K is dimensionless constant.

Now if the dimensions in mass, length and time are denoted by M, L and T the dimensional formula for

Thickness $[h] = [M^0 * L * T^0]$

Density $[\rho] = [M * L^{-3} * T^0]$

Surface tension $[\gamma] = [M * L^0 * T^{-2}]$

And Gravity $[g] = [M^0 * L * T^{-2}]$

Writing dimensional equation using equation (1) we get

$$[M^0LT^0] = [M^0L^0T^0]\,[M^1L^{-3}T^0]^a[M^1L^0T^{-2}]^b[M^0L^1T^{-2}]^c$$

Or $[M^0LT^0] = [M^0L^0T^0]\,[M^aL^{-3a}T^0][M^bL^0T^{-2b}][M^0L^cT^{-2c}]$

Or $[M^0LT^0] = [M^{a+b}\,L^{-3a+c}\,T^{-2b-2c}]$

Now as the dimensions of a fundamental quantity is same on either side of the dimensional equation, equating dimensions in M, L and T on either side of the equation we get

$a + b = 0$ --------------------------- (2)

$-3a + c = 1$ --------------------------- (3)

And $-2b - 2c = 0$ --------------------------- (4)

From equation (1) substituting 'a = - b' in equation (3) we get

$3b + c = 1$ --------------------------- (5)

Now adding equations (4) and 2*(5) we get

4*b = 2 or b = ½

Substituting in (5) we get

3/2 + c = 1 or c = - ½

From equation (1) we get a = - ½

Substituting these values in equation (1) we get

$$h = K\,\rho^{-\frac{1}{2}}\gamma^{\frac{1}{2}}g^{-\frac{1}{2}}$$

or $$h = K\,\frac{\gamma^{\frac{1}{2}}}{\rho^{\frac{1}{2}}g^{\frac{1}{2}}} = K\sqrt{\frac{\gamma}{\rho g}}$$

This is the required relation.