

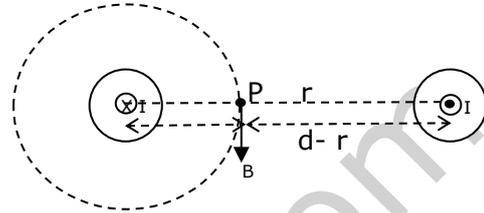
Q- Find inductance per unit length between two parallel conducting wires, each with radius  $a$ , separated by distance  $d$ , where  $d \gg a$ ?

Solution: For the inductance per unit length consider the parallel wires and let the current in the first wire is in to the paper and in the second wire it is out of the page.

Using Ampere's law to get magnetic field  $B$  at point  $P$  due to first wire we have

$$\oint \vec{B} \cdot d\vec{l} = B_r * 2\pi r = \mu_0 I$$

gives 
$$B_r = \frac{\mu_0 I}{2\pi r}$$



and similarly, the magnitude of magnetic field at  $P$  due to the second wire is given by

$$B'_r = \frac{\mu_0 I}{2\pi(d-r)}$$

As the direction of the current in the wires is in opposite direction thus the field due to both wires at point  $P$  will be in the same direction (right hand thumb rule) and hence total field at distance  $r$  from the axis of the first wire is given by

$$B = \frac{\mu_0 I}{2\pi} \left( \frac{1}{r} + \frac{1}{d-r} \right) \quad \text{----- (A)}$$

Now consider a rectangular arc of length  $l$  parallel to the wires and with infinitesimally small width  $dr$  along the line joining the axes of the wires, at a distance  $r$  from the axis of the first wire, the area of the rectangular loop will be  $l*dr$  and as the width of this rectangle is very small, the field at this area may be taken  $B$  and will be normal to the surface of the rectangle. The magnetic flux through the rectangle will be

$$d\phi_B = \vec{B} \cdot d\vec{s} = B * l * dr$$

Or 
$$d\phi_B = \frac{\mu_0 I * l}{2\pi} \left( \frac{1}{r} + \frac{1}{d-r} \right) dr$$

And hence the total magnetic flux between the two current carrying wires will be given by

$$\phi_B = \int d\phi_B = \frac{\mu_0 I * l}{2\pi} \int_a^{d-a} \left( \frac{1}{r} + \frac{1}{d-r} \right) dr$$

Or 
$$\phi_B = \frac{\mu_0 I * l}{2\pi} \left[ \int_a^{d-a} \left( \frac{1}{r} \right) dr + \int_a^{d-a} \left( \frac{1}{d-r} \right) dr \right]$$

Or 
$$\phi_B = \frac{\mu_0 I * l}{2\pi} \left[ \ln \left( \frac{d-a}{a} \right) - \ln \left( \frac{a}{d-a} \right) \right]$$

Or 
$$\phi_B = \frac{\mu_0 I * l}{\pi} \ln \left( \frac{d-a}{a} \right)$$

As the radius  $a$  of the wires is much less than the distance  $d$ , neglecting  $a$  as compared to  $d$  we get

$$\phi_B = \frac{\mu_0 I * l}{\pi} \ln\left(\frac{d}{a}\right)$$

Or the flux linked with the unit length of the line will be

$$\frac{\phi_B}{l} = \frac{\mu_0 I}{\pi} \ln\left(\frac{d}{a}\right)$$

As the inductance of a system is given by the ratio of the magnetic flux linked to the current in the system we get the inductance of the parallel line of thin wires per unit length as

$$L = \frac{\phi_B}{I * l} = \frac{\mu_0}{\pi} \ln\left(\frac{d}{a}\right)$$

Thus, the inductance per unit length of a parallel line of thin wire per unit length will be

$$L = \frac{\mu_0}{\pi} \ln\left(\frac{d}{a}\right)$$

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