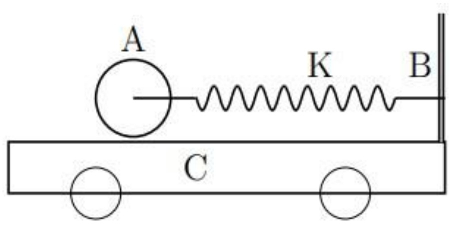

Q- A uniform cylinder A has a mass m, radius r and can roll without sliding on a cart C of mass M. cilinder A is attached to a spring AB of force constant K as in figure. If the floor is smooth and the system is released from rest when the spring is stertched by Δx,

(a) what is the velocity of the cart?

(b) what is the angular velocity of the sylinder?

Solution:

Let the velocity of the cart when the spring is just reaches undeformed position is v and the velocity of the cylinder axis at this instant is v_C,

As there is no external force acting on the system, applying the law of conservation of linear momentum we get

$m*v_C + M*v = 0$

Or $v_C = - M*v/m$ ---------------------------- (1)

(Here the negative sign shows that the cylinder axis moves opposite to the cart.)

The velocity of the cylinder axis relative to the cart will be

$$v_R = v_C - v = -\left(\frac{M}{m} + 1\right)v$$

As the cylinder rolls on the cart without slipping, its angular velocity in magnitude will be given by

$$\omega = \frac{v_R}{r} = \left(\frac{M}{m} + 1\right)\frac{v}{r}$$ -------------------- (2)

Now the total kinetic energy of the system is

KE = kinetic energy due to translation of the cart +
translational kinetic energy of the cylinder +
rotational kinetic energy of the cylinder
(floor is smooth so the wheels will not rotate)

Or KE = $\frac{1}{2}Mv^2 + \frac{1}{2}mv_C^2 + \frac{1}{2}I\omega^2$

(Where I is the moment of inertia of the cylinder given by

$I = ½ mr^2$

This total kinetic energy of the system is gained from the loss in elastic potential energy of the spring hence we can say

Gain in kinetic energy = Loss in elastic potential energy

Or $\frac{1}{2}Mv^2 + \frac{1}{2}mv_C^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}K\Delta x^2$

Or $Mv^2 + mv_C^2 + I\omega^2 = K\Delta x^2$

Substituting the values of v_C and ω from equations (1) and (2) we get

$$Mv^2 + m\left(-\frac{Mv}{m}\right)^2 + \left(\frac{1}{2}mr^2\right)\left[\left(\frac{M}{m} + 1\right)\frac{v}{r}\right]^2 = K\Delta x^2$$

Or $$\left[M + m\left(\frac{M}{m}\right)^2 + \left(\frac{1}{2}m\right)\left(\frac{M}{m} + 1\right)^2\right]v^2 = K\Delta x^2$$

Or $$v^2 = \frac{K\Delta x^2}{\left[M + \frac{M^2}{m} + \frac{(M+m)^2}{2m}\right]}$$

Or $$v = \sqrt{\frac{2m\,K}{[2M(m+M)+(M+m)^2]}}\ \Delta x$$

Or $$v = \sqrt{\frac{2m\,K}{(m+M)(3M+m)}}\ \Delta x$$

(b) And the angular velocity of the cylinder

$$\omega = \left(\frac{M}{m} + 1\right)\frac{v}{r} = \frac{(M+m)}{mr}\sqrt{\frac{2m\,K}{(m+M)(3M+m)}}\ \Delta x$$

Or $$\omega = \sqrt{\frac{2(M+m)\,K}{m(3M+m)}}\ \frac{\Delta x}{r}$$