Q- small light disc is attached with one end of a spring of constant K, the other end is fixed to ground and stands vertical. A small block of mass m is dropped from a height h above the disc. Considering elastic impact and only vertical motion find the maximum velocity and maximum acceleration of the ball in consequent motion.

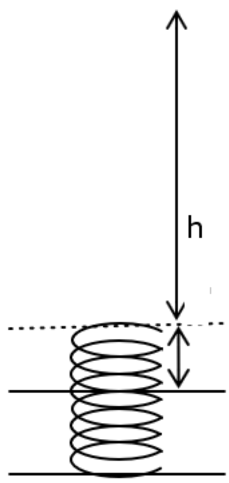
Solution:

Till the impact the block is moving with acceleration due to gravity g. After impact the spring will get compressed and apply a force in upward direction and thus the acceleration decreases but velocity will still increase. When net force on it becomes zero velocity becomes maximum because, after that spring force is greater than the gravitational force and it start decelerating.

Compression in the spring when net force is zero given by

$$k\Delta l - mg = 0$$

or $\Delta l = \frac{mg}{K}$



Velocity at this position is maximum and is given by the law of conservation of energy as

Gain in kinetic energy + gain in elastic potential energy
= loss in gravitational energy

Or $\frac{1}{2}\, m\, v_{max}^2 + \frac{1}{2}\, k\, \Delta l^2 = mg(h + \Delta l)$

Or $m\, v_{max}^2 + K\left(\frac{mg}{K}\right)^2 = 2mg\left(h + \frac{mg}{K}\right)$

Or $m\, v_{max}^2 + \frac{m^2 g^2}{K} = 2mgh + \frac{2m^2 g^2}{K}$

Or $v_{max}^2 = 2gh + \frac{mg^2}{K}$

Or $v_{max} = \sqrt{2gh + \frac{mg^2}{K}}$

Now the acceleration is maximum when the velocity of the block will become zero. If compression in the spring in this situation be ΔL then using law of conservation of energy again we get

Gain in elastic potential energy = loss in gravitational potential energy

Or $\frac{1}{2}\, K\, \Delta L^2 = mg(h + \Delta L)$

Or $K\, \Delta L^2 - 2mg\, \Delta L - 2mgh = 0$

Or $\Delta L = \frac{mg}{K} \pm \sqrt{\left(\frac{mg}{K}\right)^2 + 2\frac{mgh}{K}}$

And the acceleration will be

$$a_{max} = \frac{F}{m} = \frac{mg - K\Delta L}{m} = g - \frac{K}{m}\frac{mg}{K} \pm \left(\sqrt{\left(\frac{mg}{K}\right)^2\left(\frac{K}{m}\right)^2 + 2\frac{mgh}{K}\left(\frac{K}{m}\right)^2}\right)$$

or $a_{max} = \pm\left(\sqrt{g^2 + \frac{2ghK}{m}}\right)$